Dynamics of bubbles in liquid dielectrics under the action of an electric field: lattice Boltzmann method

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Abstract. The initial stage of deformation of bubbles before partial discharges under the action of the electric field is studied. The bubbles are placed on the surface of the electrode. The lattice Boltzmann method is used for 3D computer simulations of dielectric liquid dynamics. The electrostatic forces acting on a dielectric fluid in an electric field are taken into account as complete Helmholtz formula that includes the electrostriction forces. After the application of the electric field, the bubbles begin to elongate in the field direction. Above the critical value of the electrical Bond number, the bubbles can detach from the surface of the electrode and float.

1. Introduction

One type of partial discharges (PD) in liquid dielectrics is the electrical discharges inside small bubbles [1-3]. To simulate partial discharges in bubbles located in a liquid dielectric on the electrode surface, it is necessary not only to calculate the distribution of electric fields, but also to describe the change in the shape of bubbles under the action of an electric field. A similar problem related to the behavior of bubbles arises in simulating the boiling of liquid in the presence of an electric field. Such a problem was modeled in [4] in the two-dimensional case.

In this paper, a three-dimensional computer-based electrohydrodynamic model of the dynamics of vapor bubbles in dielectric liquids is developed to take into account surface tension and processes of evaporation and condensation at the liquid-vapor interfaces. For this purpose, the lattice Boltzmann method is used. At this stage of the research, the calculations were carried out without the redistribution of free electrical charges in the bubbles due to the currents during partial discharges.

2. Electrostatic forces and electric field calculations

The electrostatic forces acting on nonuniform dielectrics consist of the action of the electric field on polarization charges and the electrostriction forces [5]

$$\mathbf{F} = -\frac{\varepsilon_0 E^2}{2}\nabla\varepsilon + \frac{\varepsilon_0}{2}\nabla\left[E^2\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right]. \quad (1)$$

Here, ε_0 is the dielectric constant of free space, ε is the dielectric permittivity of fluid, E is the value of an electric field and ρ is the density of fluid.

To take into account the effect of electrical body forces, together with the hydrodynamic flows, it is necessary to simultaneously solve the problem of the distribution of the electric field in a non-uniform

dielectric. The calculation of the electric field potential $\varphi(\mathbf{x},t)$ between flat parallel electrodes placed at the distance d is performed at each time step by the method of iterations according to a finite difference scheme with central approximation of the second derivatives. The values $\varphi = 0$ at the lower electrode and $\varphi = V$ at the upper electrode are used as the boundary conditions for the electric field potential. In the x and y directions perpendicular to the electric field, the periodic boundary conditions are used. In this case, the changes in the distribution of the dielectric constant $\varepsilon(\mathbf{x},t)$ of the dielectric fluid in space and in time are taken into account. We use the Clausius–Mossotti relation [6] for the dielectric permittivity of a non-polar fluid

$$\varepsilon = 1 + \frac{3\alpha\rho}{1-\alpha\rho}, \tag{2}$$

where α is the polarizability.

The electric field is then calculated using the formula $\mathbf{E}(\mathbf{x},t) = -\nabla\varphi(\mathbf{x},t)$.

3. Lattice Boltzmann method for hydrodynamic part of simulations

To simulate the dynamics of a liquid dielectric with bubbles, we use the lattice Boltzmann equations method (LBM) that is based on the method of characteristics for the Boltzmann equation with a discrete set of velocities $\{\mathbf{c}_k\}$, where $k = 0,...,18$ for the three-dimensional lattice D3Q19. The main variables are the distribution functions $N_k(\mathbf{x},t)$ in each node of the 3D lattice. Their values are transferred to the neighbor nodes and undergo collisions in accordance with the equation

$$N_k(\mathbf{x}+\mathbf{c}_k\Delta t, t+\Delta t) = N_k(\mathbf{x},t) + \Omega_k(\{N_k(\mathbf{x},t)\}) + \Delta N_k. \tag{3}$$

Here, $\Omega_k = (N_k^{eq}(\rho,\mathbf{u}) - N_k(\mathbf{x},t))/\tau$ is the collision operator in the BGK form [7] and ΔN_k is the change of the distribution functions due to the action of body forces. The density ρ and the velocity $\mathbf{u}$ of a fluid are calculated in accordance with the equations

$$\rho = \sum_{k=0}^{b} N_k \quad \text{and} \quad \rho\mathbf{u} = \sum_{k=1}^{b} \mathbf{c}_k N_k \ . \tag{4}$$

We use the variant of LBM proposed by the authors in [8,9] that takes into account the liquid-vapor phase transitions. For this purpose the internal forces between nodes were introduced. These forces are expressed as the gradient of the pseudopotential $U = P(\rho,T) - \rho\theta$ [10]. The isotropic hybrid finite-difference approximation for gradient operator proposed in [8] is applied. We use the Exact difference method to implement the body force term in LBM [9,11]. The van der Waals equation of state written in the reduced variables is used for the fluid

$$P = \frac{8\rho T}{3-\rho} - 3\rho^2. \tag{5}$$

For the hydrodynamic problem, no-slip boundary conditions are fulfilled on the electrodes. The periodic boundary conditions are used in x and y directions. The interaction between a fluid and solid surfaces is introduced as forces between a fluid node $\mathbf{x}$ and the nearest five nodes on the solid surface (figure 1)

$$\mathbf{F}(\mathbf{x}) = B\Phi(\mathbf{x}) \sum_{i=1}^{5} w(\mathbf{e}_i)\Phi_{\text{solid}}(\mathbf{x}+\mathbf{e}_i)\cdot\mathbf{e}_i \ . \tag{6}$$

Here, the function $\Phi(\mathbf{x}) = \sqrt{\rho\theta - P(\rho,T)}$. The value of the function Φ_{solid} at a node of electrode takes the same value as at the nearest node of fluid. The parameter B allows one to change the wettability of

the solid surface. The parameter B in (6) should be equal to 1 for the value of contact angle of bubbles equal to 90º on the surface of solid electrode.

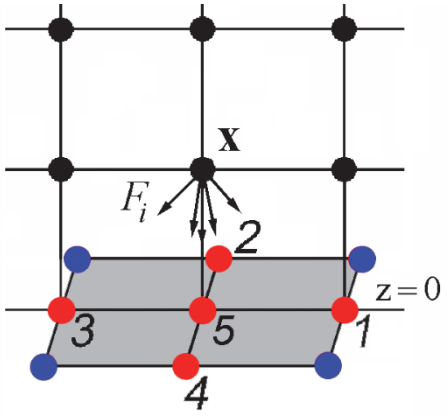

Figure 1. Interaction forces that act on a fluid node **x** from the nearest five nodes at the electrode.

The calculations are performed on two multi-processor graphics cards NVIDIA Titan-Xp (each has 3840 cores and 12 GB of fast internal memory) using the CUDA (Compute Unified Device Architecture) technology.

4. Simulation results

Figures 2 and 3 show the dynamics of an initially hemispherical bubble of a radius of 50 grid nodes under the action of an electric field. The bubble is located on the surface of the lower electrode. The value of the contact angle is set at 90°.

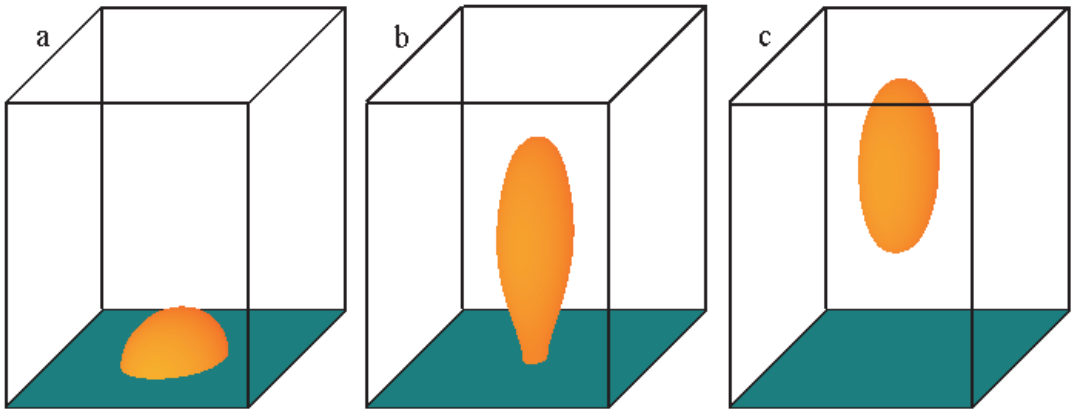

Figure 2. Three-dimensional simulation of the dynamics of an initially hemispherical vapor bubble placed on the electrode in the electric field. Lattice size is 256×256×320. $R_0 = 50$, Bo = 0.67, $\varepsilon_l = 2.3$. $t = 0$ (a), 6600 (b), 11200 (c).

The bubble begins to elongate (figure 3b). The increase in the volume of the bubble is due to the evaporation of a liquid into the bubble, which is stretched by electrostatic forces. After elongation of the bubble, it can detach from the surface of the lower electrode. Then, the bubble acquires an approximately ellipsoidal shape due to the electric field and begins to float up due to the presence of a gravity field g. The value of the Bond number $\mathrm{Bo} = \rho g R_0^2 / \sigma$ is equal to 0.67. The electrical Bond number has the form

$$\mathrm{Bo}_e = \frac{\varepsilon_0 E^2}{2}(\varepsilon_l - 1)\frac{R_0}{2\sigma}. \quad (7)$$

Here, ρ is the density of the liquid, ε_l is the dielectric permittivity of the liquid, $E = V/d$ is the mean value of the electric field in the gap, R_0 is the initial radius of a bubble and σ is the surface tension. For this simulation, the electrical Bond number $\mathrm{Bo}_e = 8.5$. In physical units, this value corresponds to $E \sim 40$ KV/cm for $R_0 = 5$ mm and $\sigma \approx 0.03$ N/m.

The kinematic viscosity of the liquid ν corresponds to the value of the Ohnesorge number $\mathrm{Oh} = \nu\sqrt{\rho/(\sigma R_0)} = 0.04$.

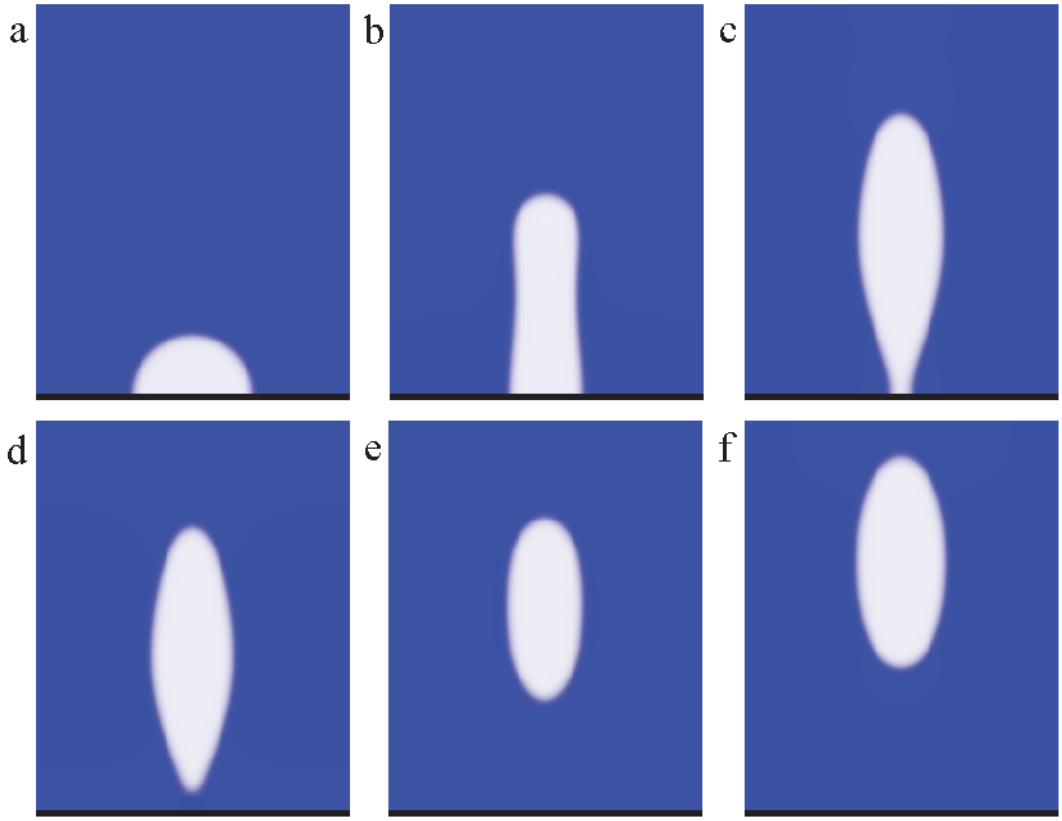

Figure 3. Dynamics of initially hemispherical vapor bubble on the electrode in electric field (central $x-z$ cross section). $R_0 = 50$, Bo = 0.67, $\varepsilon_l = 2.3$. Lattice size is 256×256×320. $t = 0$ (a), 5000 (b), 6600 (c), 6800 (d), 9600 (e), 11200 (f).

Figure 4 shows the dynamics of two initially hemispherical bubbles of different sizes (40 and 50 grid nodes) located on the surface of the lower electrode. Since the process occurs in a "closed" volume, and the liquid is almost incompressible, an increase in the volume of a large bubble on which electrostatic forces act more strongly and consequently a decrease in the volume of a small one occur. After a while, the small bubble dissolves completely by condensation of the vapor on the liquid surface. This process is facilitated by the increased pressure inside the small bubble due to surface tension and also a shift in phase equilibrium due to negative curvature. The Ohnesorge number is approximately equal to 0.14. The electrical Bond number is about 12.

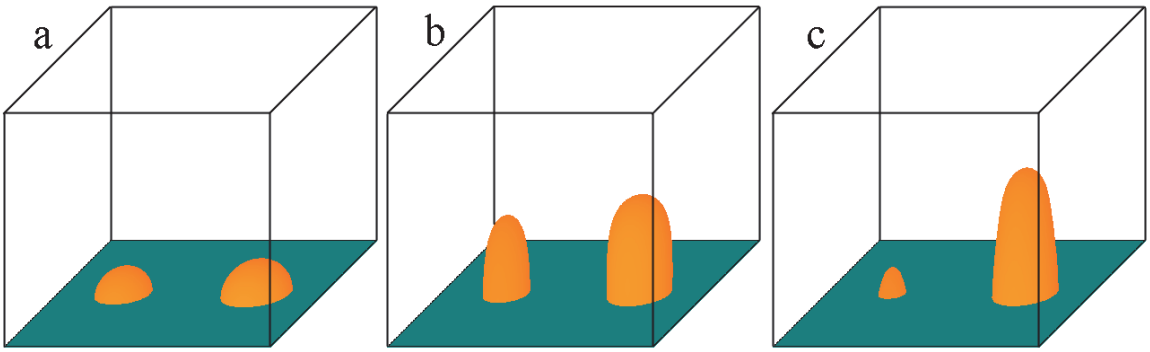

Figure 4. Dynamics of two vapor bubbles of different sizes under the action of the electric field. $\varepsilon_l = 2.3$. Lattice size is 400×400×352. $t = 0$ (a), 2400 (b), 3600 (c).

Conclusion

The initially hemispherical bubbles located on the surface of the lower electrode begin to elongate under the action of the electric field. Above the critical value of the initial electrical Bond number, the bubbles can detach from the surface of the electrode and begin to float upward. During the elongation of bubbles, the conditions for the Paschen's law can be satisfied, and partial discharges in the bubbles can be originated. After this stage, it is necessary to take into account the transfer of free charges in bubbles during the partial discharges.

The method of lattice Boltzmann equations is a convenient tool for simulating electrohydrodynamic problems related to the behavior of bubbles in electric fields. The calculated size and the magnitude of deformation of the bubbles in electric fields can be used to study the conditions for partial discharges inception in vapor-gas bubbles.

Acknowledgements

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References

[1] Kreuger F H, Gulski E and Krivda A 1993 *IEEE Trans. Electr. Insul.* **28** 917–31
[2] Niemeyer L 1995 *IEEE Trans. Dielectr. Electr. Insul.* **2** 510–28
[3] Kupershtokh A L and Karpov D I 2018 *J. Electrostatics* **94** 8–13
[4] Feng Y, Li H, Guo K, Lei X and Zhao J 2019 *Int. J. Heat and Mass Transfer* **135** 885–96
[5] Landau L D and Lifshitz E M 1959 *Electrodynamics of Continuous Media* (Oxford: Pergamon Press)
[6] Feynman R P, Leighton R B and Sands M 1964 *The Feynman Lectures on Physics* (Massachusetts, London: Addison-Wesley) **2**
[7] Bhatnagar P L, Gross E P and Krook M K 1954 *Phys. Rev.* **94** (3) 511–25
[8] Kupershtokh A L, Medvedev D A and Karpov D I 2009 *Computers and Mathematics with Applications* **58** (5) 965–74
[9] Kupershtokh A L 2010 *Computers and Mathematics with Applications* **59** (7) 2236–45
[10] Qian Y H and Chen S 1997 *Int. J. Mod. Phys.* **8** (4) 763–71
[11] Kupershtokh A L 2004 *Proc. 5th International EHD Workshop (Poitiers)* (Poitiers: Universite de Poitier, France) pp 241–6