STOCHASTIC MODELING OF LIGHTNING PROCESS

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Abstract: In the present work, a stochastic model is proposed for simulation of lightning process. Two different states of a conductive structure that correspond to streamer and leader channels are introduced. The model can be used, in principle, for determination of a protective zone of lightning arresters and for calculation of a probability of damage of tall structures, transmission lines, etc.

1. Introduction

The normal earth flash starts somewhere in a negative charge center in the lower region of the thundercloud and propagates towards the ground in the form of a stepped leader. The direction of each step is likely determined by the local electric-field distribution around the tip of discharge channel.

The lightning can be considered as a breakdown in air between two "electrodes", the cloud and the earth. For simulation of breakdown in dielectrics, several authors have developed the so-called stochastic models [1-6]. Stochastic models are based on the assumption that the probability of the conductive structure growth inside a dielectric is related to a function of the local electric field as $p \sim r(E)$.

Most of these models have been developed for simulation of the breakdown that occurs in the form of streamer channels. Many authors considered the conductive structure as equipotential [1, 2], that allowed them to solve the Laplace equation for calculation of electric field.

However, a simulation of the breakdown in long air gaps and, especially, a simulation of a lightning are not possible using the existing stochastic models. The physical reason is that the lightning growth occurs in the form of streamer-leader system. As a consequence the tips of lightning channels can grow in low electric field by reproducing self-sustained conditions during tips propagation.

For simulation of the lightning process, a new stochastic model was developed [7], taking into account the existence of two different conductivity phases. A highly conductive phase corresponds to leader and a lowly conductive phase corresponds to streamer. The main scope of our paper is to obtain some qualitative results and to show that this new stochastic model can be used as a basis for development of more detailed models for description of lightning growth.

2. The Model

We specially considered a problem in two-dimensional case. This approach is well-taken in situation if geometry of electric field plays only an auxiliary role. Really, all of conductive tree growth criteria in existent models of lightning process are qualitative till now and are needed in considerable improvements. So, it is not necessary to calculate the electric field more adequately. This approach is usual for such kind of simulations.

The existence of two different states of channel conductivity during the simulation requires separate consideration of the growth of streamer channels and the transition of streamers to leader. For the growth of the streamers, two criteria were used.

The first named field fluctuation criterion (FFC) [4, 5] was initially developed for the simulation of the streamer growth inside liquid dielectrics. The FFC is a multi-element model. This means that several new bonds can be added to the conductive structure at each time step. A new streamer segment is added to the structure if

$$E_i > E_* - \delta . \qquad (1)$$

Here E_i is the mean local electric field near lattice point. The parameter E_* is a characteristic parameter of the dielectric and depends on typical values of the humidity and air density. A random value δ is assumed to take into account uncertainties of atmospheric conditions, initial ionization, inhomogeneities in air, thermal and other fluctuations, including fluctuations of local microfields acting on the molecules.

The probability distribution for fluctuations δ is the following

$$f(\delta) = \frac{\exp(-\delta / g)}{g}, \qquad (2)$$

that is $\delta = -g \ln \xi$. Here ξ is a random number that is uniformly distributed in the interval from 0 to 1. Large fluctuations are statistically rare and occur in the far wing of the distribution.

The definite time of appearance of the i-th bond corresponding to each random value δ_i is equal to

$$\tau_i = -\frac{\ln(1-\exp(-\delta_i / g))}{r(E_i)}. \qquad (3)$$

Thus, the condition (1) for the growth of a new streamer segment is equivalent to the inequality

$$\tau_i < \tau \qquad (4)$$

where τ is the physical time interval [5, 7].

Usually, the time step τ is short and the probability of appearance of new streamer segment is small enough. In this case, the function $r(E)$ has the form

$$r(E) \approx A e^{E/g} \text{ where } A = \frac{1}{\tau} e^{-E_*/g}. \qquad (5)$$

The second criterion was the so-called multi element stochastic time lag (MESTL) proposed in [5]. In this multi-element model the physical time step τ is chosen arbitrary. The delay time of appearance of the i-th bond is calculated for all candidate bonds

$$\tau_i = -\ln(\xi_i) \Big/ r(E_i). \qquad (6)$$

All possible bonds arise for that τ_i less than τ. For this criterion we used $r(E) \approx (E/E_*)^n$.

The next point of our model is the transition of the streamers to leader. This transition is a complex physical phenomenon and many theories have been proposed for its description [8-10]. For the transformation of a streamer to a highly conductive arc, it seems to be of importance the energy input inside a streamer segment. If the energy released due to the current flow inside streamer filaments achieve certain critical value, the streamer to leader transition occurs.

The total energy released inside a segment of streamer filament by the time t was calculated using the following formula

$$W_i = h \cdot S \cdot \sigma \cdot \int_{t_i}^{t} E_i^2 dt \qquad (7)$$

where h is the length of the streamer segment, S is the cross section, σ is the conductivity of the streamer, and t_i is the time when arose the i-th bond. If the released energy is greater than a certain critical value

$$W_i > W_* , \qquad (8)$$

the streamer segment converts into leader.

In many cases as the leader approaches the ground a return stroke is originated, especially from tall objects on the lower surface, because of the steep increase of the electric field.

3. Results

The simulations were carried out in a two dimensional rectangular area with lattices up to 150×150. During the simulation, the lattice spacing was assumed to be equal to 3 m.

The leader channel was considered to be equipotential due to its high conductivity. On the other hand, it was supposed that the streamers do not influence significantly the distribution of the electric filed. Thus, the electric-field potential φ is calculated by solving the Laplace equation with boundary conditions on the electrodes and the leader channel structure

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0. \qquad (9)$$

The upper electrode corresponds to the point of the cloud where the lightning is initiated. At each time step, new streamer channels may arise ahead the leader tip while some of the existing streamers may convert to new leader segment. This process continues until the leader reaches the ground.

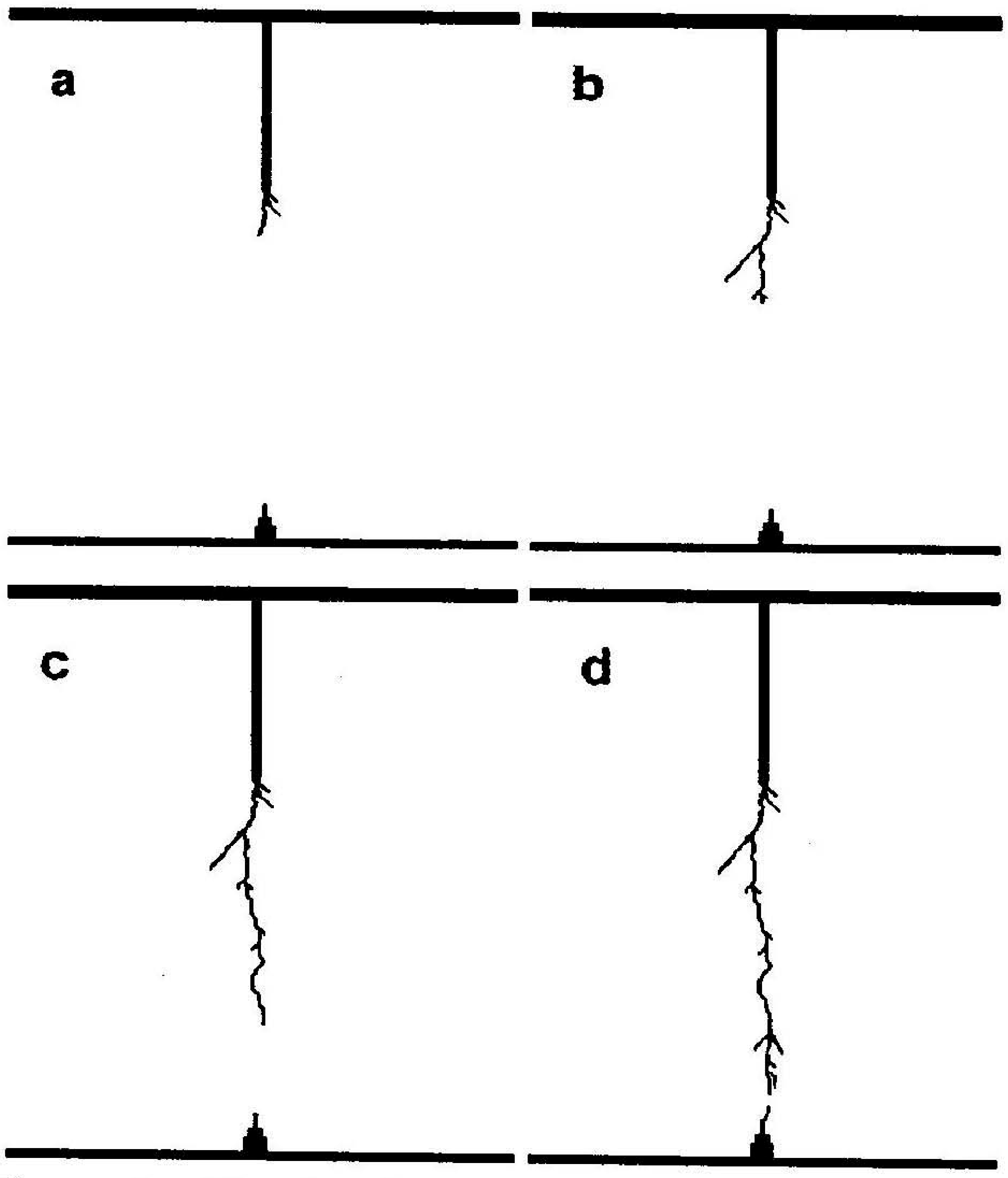

Figure 1. The development of the stepped leader in time (four different instants).

In **fig. 1** the development of a stepped leader is illustrated in four different time moments. In **fig.1d** one can see the return stroke arising from the tall building.

The most well known and least expensive form of shielding devices is the lightning rod or air terminal. The lightning rods can protect buildings inside a specific area around them, by intercepting the lightning stroke. With the lightning rod, a circuit of least resistance is provided to the upward spark and thereby establishes the final segment of the cloud to earth discharge.

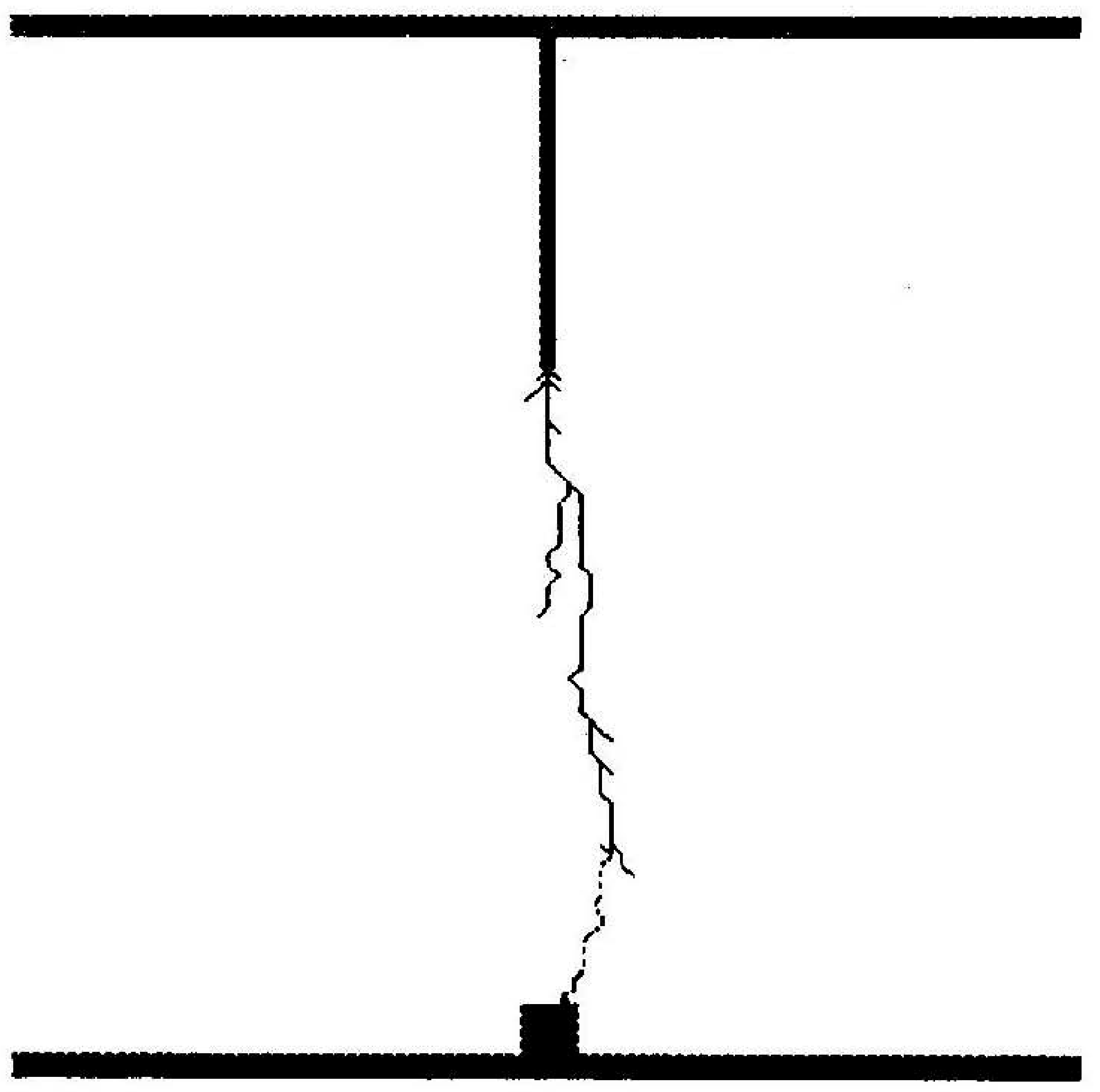

Figure 2. Lightning stroke to a building.

A lightning hitting a building on the ground is shown in **fig. 2**. In **fig. 3** the same building is protected by an air terminal which intercepts the lightning stroke. After several simulations one can determine the probability of a lightning stroke to damage the building in accordance to the position of the rod, its height and the place of the lightning origin.

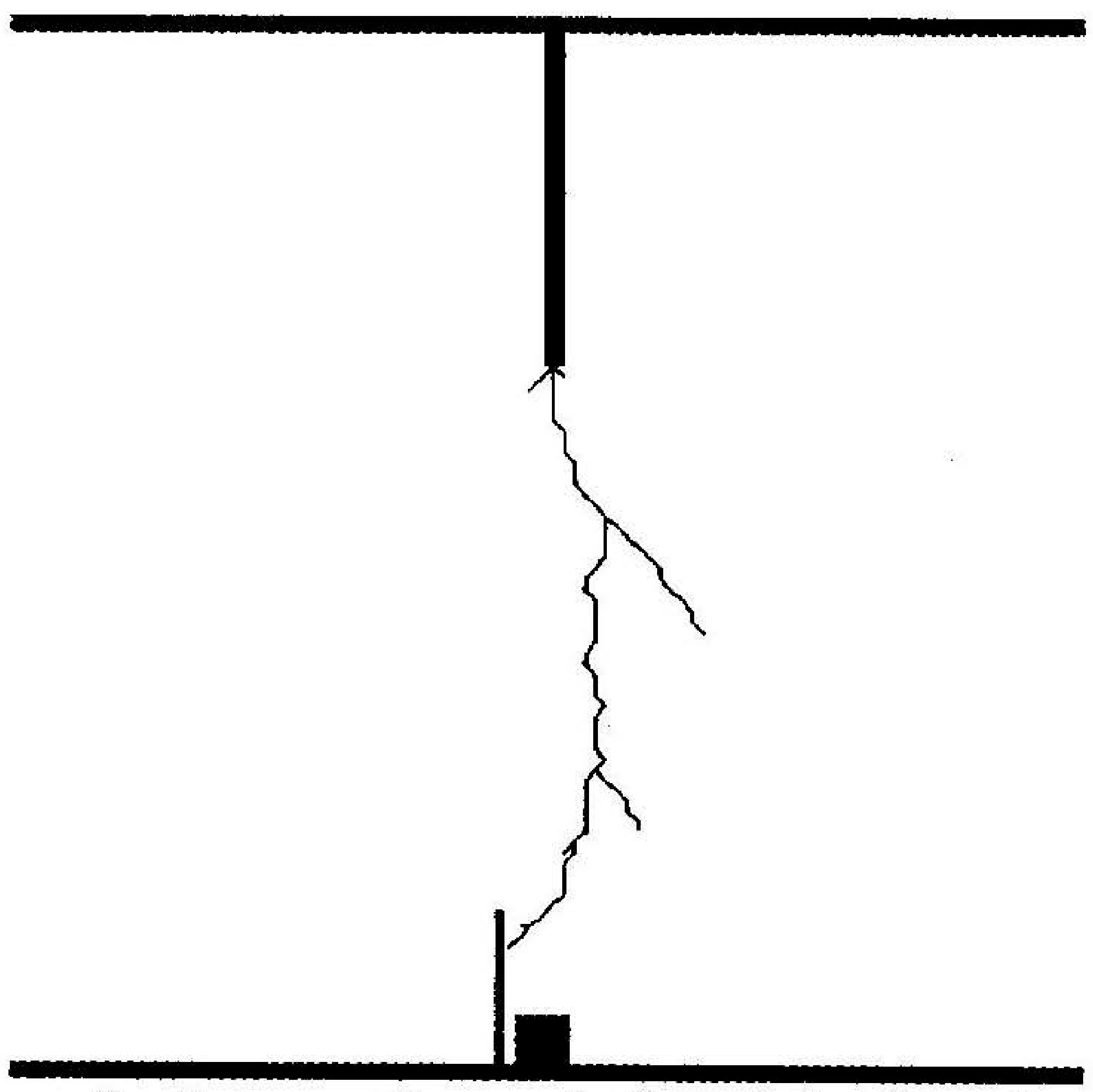

Figure 3. Interception of the lightning by an air terminal.

The existence of a lightning rod on the ground resulted in a significant redistribution of electric field at the lower surface. The distribution of the lightning strokes without the rod on the lower surface is illustrated in **figures 4a** and **5a**, for the FFC and MESTL models respectively. The distributions are close to normal in both cases. The presence of a lightning rod in the center of the lower plane results in the redistribution of the positions of the lightning strokes (**figures 4b, 5b**).

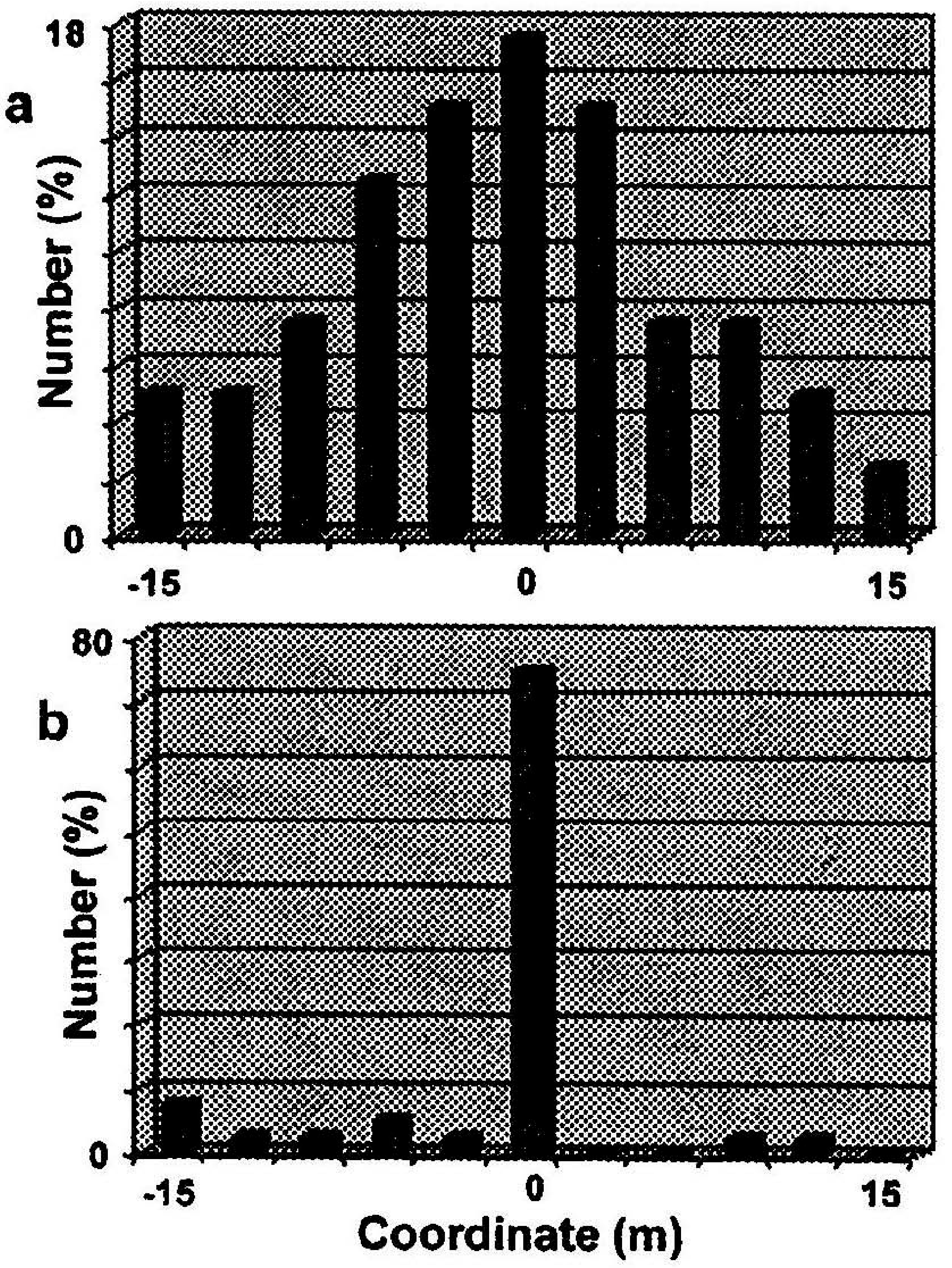

Figure 4. The distribution of the lightning strokes for FFC model. A lightning rod is absent (a) and a lightning rod is placed at the center of the plane (b).

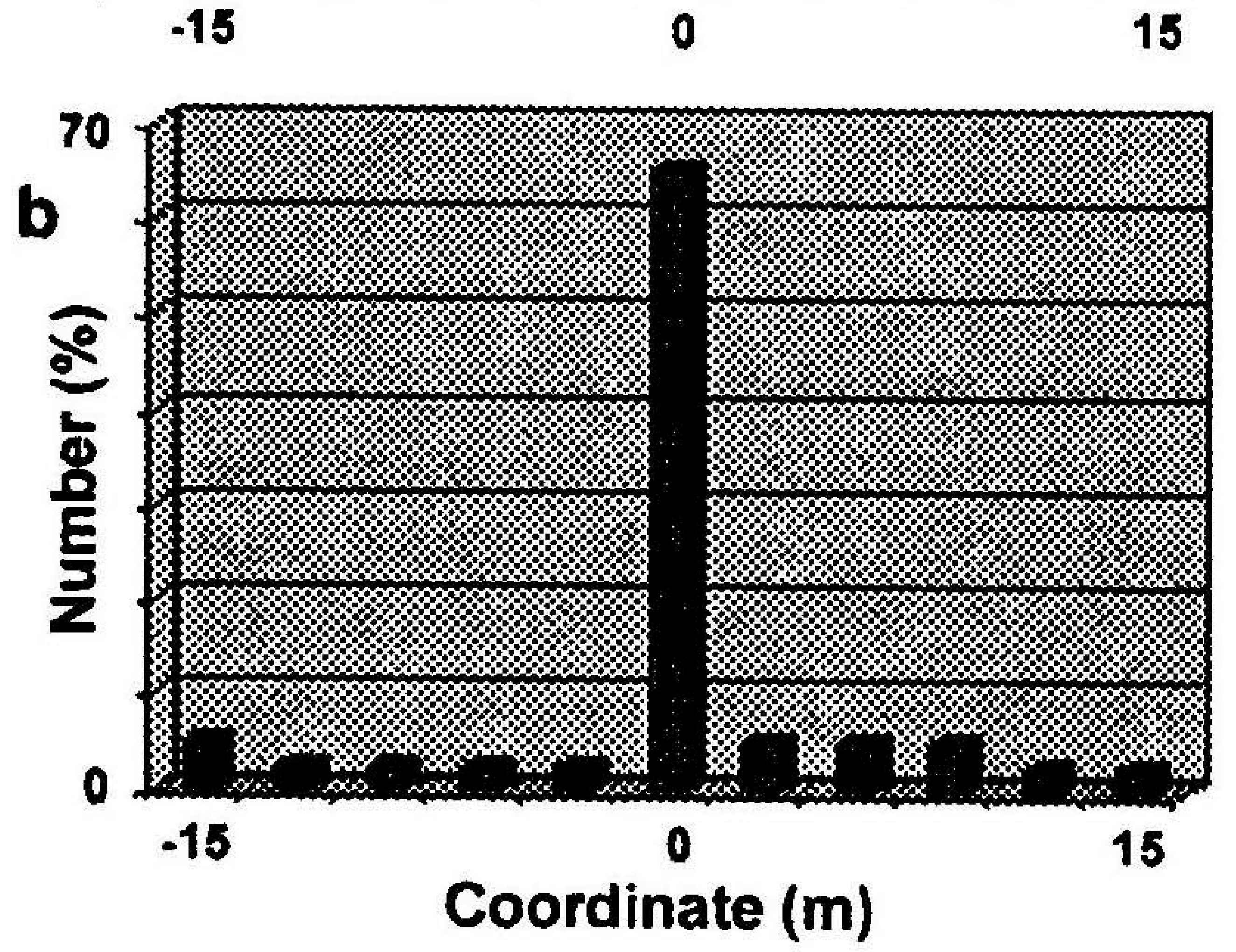

Figure 5. The distribution of the lightning strokes for MESTL model. A lightning rod is absent (a) and a lightning rod is placed at the center of the plane (b).

The number of trials was up to 40 for each of four different cases. For more accurate results, a significantly larger number of trials is required. All simulations were carried out using arbitrary units for the time and applied voltage. The values of the parameters for the FFC model were $E_* = 1$ and $g = 0.08$. For the MESTL model, we used slower increasing function $r(E)$ at the values of parameters $E_* = 1$ and $n = 3$. The mean value of electric field in the gap $E_0 = V/d$ was equal to 0.25 where V is the applied voltage and d is the gap length.

4. Conclusions

Our results qualitatively agree well with the results of work [11], in which the conductivity of the channels was assumed to be proportional to the internal energy of plasma.

The results obtained with FFC model do not differ qualitatively from the results obtained with MESTL model, provided that the function $r(E)$ was the same. The equivalence of these two criteria of conductive tree growth was shown in [5].

The model proposed in this paper, can be used as a basis for the development of more efficient models. The main purpose of these models is the reliable determination of the protective zones of the air terminals and the calculation of the probability of damage of tall structures and transmission lines. The models can also be used for the determination of the fifty percent breakdown voltage in the case of long air gaps under an impulse voltage.

The proposed stochastic model have many capabilities of further development in order to describe more adequately the physical process of the lightning growth. For example, it is possible to take into account an approximately constant voltage drop along the leader channels of order 10 kV/cm that was observed in experiments. It is needed also to introduce into the model the real scales for voltage, time and space using reliable experimental data.

It is also of importance the determination of the critical energy W_* required for the transition of the streamer to leader phase.

A more complex model will be if we introduce a conductivity along the leader channels and consider how the presence of the space charge influence the distribution of the electric field, that is taken into account in the present model by indirection. In this case, it is necessary to solve Poisson's equation together with the equation of electric charge flow along the branches of conductive structure [5].

As mentioned above, the distribution of electric-field potential was calculated in two-dimensional geometry. After all these improvements of the model, the three-dimensional calculations of the electric field will become the most preferable.

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