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"Relay-race" mechanism of partial discharges in a long chain of cavities for stochastic nature of process

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ABSTRACT

A stochastic model of partial discharges (PDs) inside a linear chain of gas cavities in condensed dielectrics is developed. The equations for electric field potential and electric charge transfer are solved together for dielectric with these inclusions. Computer simulations show the possibility of a "relay-race" mechanism of propagation of partial discharges in this chain of gas cavities even if the stochastic nature of partial discharge is taken into account. This mechanism can be realized if the distances between cavities are small enough and the dependence of probability function of partial discharges $r(E)$ on electric field strength is rather sharp. In this case, the wave of partial discharges can propagate along the chain of cavities. The PD waves can be initiated in the first cavity as well as in the last cavity in the chain. Occasionally, two PD waves can arise from the both edges of the chain. The sequence of partial discharges in the inclusions has a completely stochastic character for a weak mutual influence of cavities on each other.

1. Introduction

When a high voltage is applied to an interelectrode gap with a condensed dielectrics, the pulses of the electric current can be registered in the external circuit (partial discharges). The partial discharges (PDs) do not lead to the loss of the insulating properties of the dielectric as a whole. The frequency and magnitudes of PDs can be used to predict the possibility of a breakdown of the interelectrode gap. One type of partial discharges in condensed dielectrics is the electrical discharges inside small gas inclusions (voids in solid or bubbles in liquid dielectrics) [1,2].

The breakdown strength of gas inside the small cavities in solid dielectrics or inside the microbubbles in a liquid is much lower than that of the condensed matter. To start an ionization avalanche inside a cavity, it is necessary the sufficient value of electric field strength inside a cavity. This determines the threshold character of the partial discharge. The threshold character of electrical breakdown of dielectrics is well known and was observed in numerous studies [3–7]. Hence, to simulate the partial discharges, their threshold features should be taken into account. Moreover, the stochastic behavior of electrical breakdowns and partial discharges that is caused by appearance of initial electrons must be also taken into account.

To describe the main features of partial discharges, the method of equivalent electric circuit for a cavity was proposed in Ref. [8]. One of the first attempts to introduce the stochastic nature of PDs into this method was realized in Ref. [9]. Later, this approach was developed in Refs. [3,10,11]. However, all these studies did not take into account the spatio-temporal evolution of the electric field strength in the gap. Moreover, the classical capacitance model is practically insensitive to the position of a cavity inside the gap. The works [12–15] were devoted to the computer simulations of partial discharges in liquid and solid dielectrics where the electric field distribution was calculated at every time step. The partial discharges in coupled cavities (with close distance between them along an electric field line) were simulated in Refs. [13,14]. After the discharge in one cavity, the electric field strength inside the neighbor cavity sharply increased. Therefore, the increase of the probability of PD in the neighbor cavity was demonstrated.

In the present paper, the possibility of the propagation of a PD wave along a chain of gas inclusions is studied. Such chains of bubbles may occur at the boundaries of layers in a multilayer paper insulation. The chains of bubbles also arose during the decay of cylindrical channels of an incomplete electrical discharge after the termination of the previous voltage pulse [16].

A stochastic model of partial discharges (PDs) inside a linear chain of gas cavities in condensed dielectrics is developed. The partial discharges are studied in condensed dielectrics that fills the space between two flat electrodes. Two-dimensional computer simulations of partial discharges in a linear chain of such gas inclusions located in a condensed dielectric are carried out. The inclusions are placed along the electric field line at equally close distances from each other.

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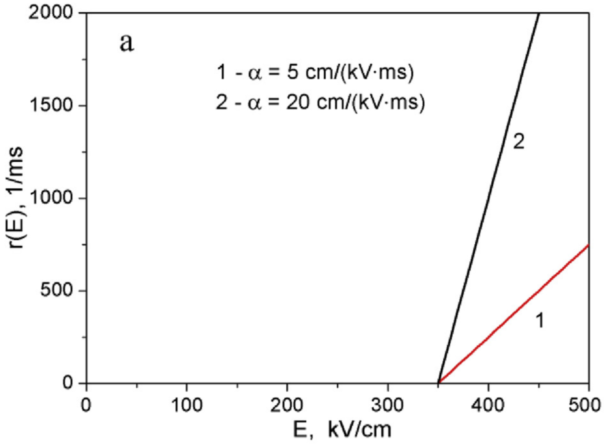

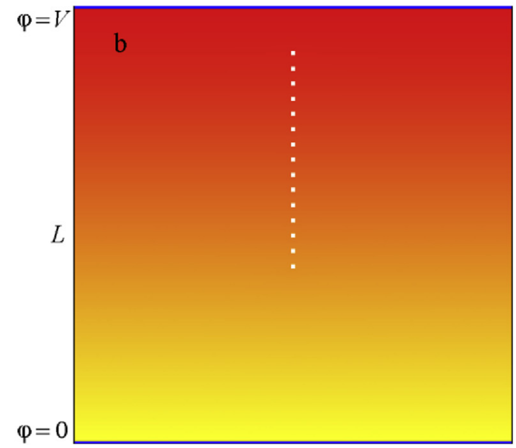

Fig. 1. (a) Probability function $r(E)$ for PD in a cavity. Curves 1 and 2 correspond to α= 5 and 20 cm/(kV·ms). (b) Chain of gas cavities in the solid dielectric. $N = 15$. L= 1 mm $\Delta y = 6h$.

The possibility of the wave of PDs that propagate along the chain of gas inclusions is demonstrated. In this case, the partial discharges occur sequentially in the inclusions one by one along the chain of these insulation defects. This interesting phenomenon can be named as a “relay-race” mechanism of propagation of partial discharges in the chain. For the “relay-race" mechanism, the streamers occur only inside the gas filled cavities but do not propagate in the condensed phase. There are several possible ways that can be used to observe this “relay-race" mechanism in experiments. The purpose of this article is to attract the attention of experimentalists to this phenomenon. Moreover, it is most important that the “relay-race” mechanism of propagation can be realized even if the stochastic nature of the phenomenon is taken into account. This mechanism differs from deterministic “hopping spread streamers” [17] that was simulated for three “bubbles” placed along electric field line in the condensed dielectrics.

2. Criterion of partial discharge in a gas cavity placed into condensed dielectrics

At the same size of gas cavities in the bulk of the dielectric and at the same gas pressure inside them, the probability of micro-breakdowns inside the inclusions depends on the local electric field within them E_i.

In 1993 Biller [18] proposed a first stochastic criterion for streamer growth based on the idea of stochastic lag times t_i. The density distribution function was used for the probability of rare events

$$F(t_i) = r(E_i)\exp(-r(E_i)t_i). \tag{1}$$

Here, $r(E)$ is the sharply increasing function on the electric field strength. The random values of the stochastic lag times t_i can be calculated in accordance with the formula

$$t_i = -\ln(\xi_i)/r(E_i), \tag{2}$$

that is equivalent to the distribution function (1). Here ξ is a random number uniformly distributed in the interval from 0 to 1. The new segment of streamer structure was generated for which the stochastic lag time was minimal. This criterion was the first single-element criterion with physical time.

For streamer structures growth, the stochastic criterion MESTL (multi-element stochastic time lag) was proposed in Refs. [19,20]. Later we used this criterion to describe the occurrence of micro-discharges in gas cavities [13,14,21]. For all nonconducting cavities, the values of stochastic lag times (2) were calculated. The stochastic criterion MESTL assumes that the micro-discharges occur during the current time step Δt in all cavities for which the conditions $t_i < \Delta t$ are satisfied.

The function $r(E)$ depends on the local electric field inside a gas cavity. For small time step $\Delta t << 1/r(E)$, the probability of a micro-discharge in a cavity is approximately equal to $f \approx r(E)\Delta t$. The typical value of time step in simulations is chosen equal to $\Delta t = 10^{-4}$ms for which the probability is small $f << 1$.

When a linearly rising voltage is applied to a discharge gap, a scatter in discharge inception voltage due to the statistical time lag is observed. In works [7,22], it was obtained that the probability of the discharge initiation is proportional to overvoltage $\Delta V = (V - V_*)$, where V_* is the critical value of the breakdown voltage of a gap. Here, we also assume that the probability of partial discharge is proportional to overvoltage ΔV for cavities of equal size.

The function $r(E)$ that describes the threshold character of partial discharges has the form [7].

$$r(E) = \begin{cases} 0 & \text{for} \quad E \le E_*, \\ \alpha(E - E_*) & \text{for} \quad E > E_*. \end{cases} \tag{3}$$

Here, E_* is the threshold value of electric field above which PDs in the gas inclusions are possible. The coefficient α is the slope of the function $r(E)$. The probability of breakdown increases sharply in a narrow range of the electric field at $E > E_*$ [4]. The threshold breakdown voltage V_* of air in a void of size d~10 μm at pressure 1 atm is approximately equal to 350 V [6], which corresponds to the threshold field $E_* \approx 350$ kV/cm. The functions $r(E)$ are shown in Fig. 1a for different values of α.

During the microdischarge inside a cavity, the electric field decreases there. If its value becomes less than some critical value E_{cr}, the energy release reduces and become small in comparison with the energy loss. Hence, a complete decay of plasma inside cavity occurs, and the microdischarge terminates. We assume that the conductivity after this moment becomes equal to zero.

3. Electric field values in gas cavities spaced in condensed dielectrics

The PDs in a chain of several identical gas inclusions equally spaced along the electric field line in condensed dielectric between two flat electrodes is studied (Fig. 1b). For 2D simulations, we use the lattice size 200 × 200 (40000 nodes). The distances between all adjacent inclusions are equal to Δy.

The form of cavity and conductive structure arising in a cavity that we used in simulations are shown in Fig. 2 [13]. Cavity size is $2h \times 2h$. Here, h is the computational lattice spacing. If the value of projection of electric field strength on a bond between nodes in gas phase become greater than critical value E_*, the all structure become conductive. As the first approximation, the conductivity of the elements of the conductive structure is assumed to be equal to a constant value σ during the short period of a partial discharge in a cavity.

The distribution of the electric field strength in the whole region

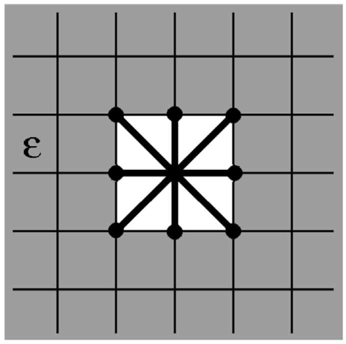

Fig. 2. The pattern of conductive structure arising in a cavity during a partial discharge that we used in simulations.

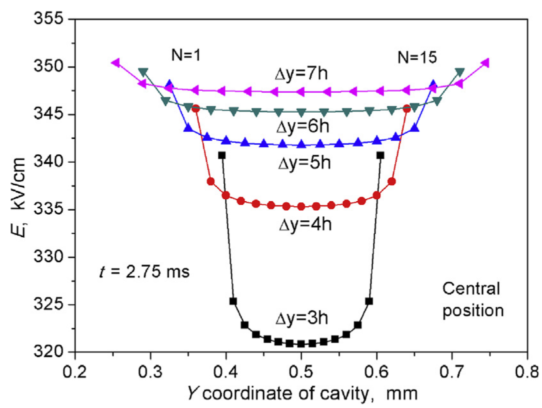

Fig. 3. The values of electric field inside gas cavities before the partial discharges (t=2.75 ms) at different distances between cavities $\Delta y = 3h,\ 4h,\ 5h,\ 6h,\ 7h$. $N = 15$. $L = 1$ mm. $E_* = 350$ kV/cm. h=5 μm α=20 cm/(kV·ms). Lattice size is 200 × 200.

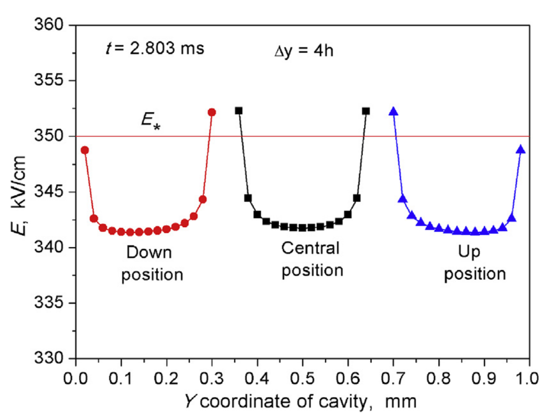

Fig. 4. The values of electric field inside gas cavities before the partial discharges (t=2.803 ms) at different position of the chains ($N = 15$). $L = 1$ mm. $E_* = 350$ kV/cm. $\Delta y = 4h$. h=5 μm α=20 cm/(kV·ms). Lattice size is 200 × 200.

between the flat electrodes (Fig. 1b) is calculated numerically at each time step by solving the Poisson's equation for potential of electric field ϕ together with the equation of the electric charge transfer inside inclusions that takes place during the partial discharges [13,14].

$$\mathrm{div}(\varepsilon\varepsilon_0 \nabla\phi) = -q, \tag{4}$$

$$\frac{\partial q}{\partial t} = -div\,\mathbf{j}, \tag{5}$$

Here, ε is the relative dielectric permittivity, q is the electric charge density. The distribution of electric field strength is defined by the equation $\mathbf{E} = -\nabla\phi$. It is assumed that the conductivity σ and the

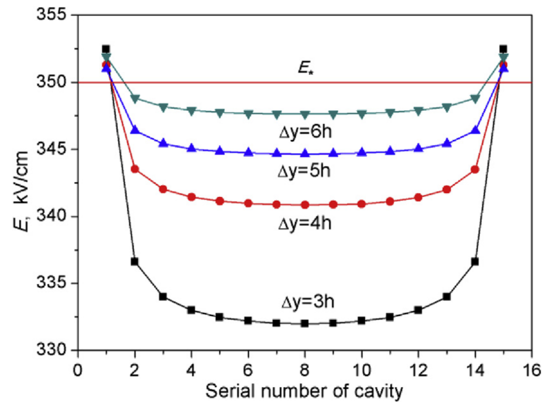

Fig. 5. The values of electric field inside all gas cavities just before the first partial discharges in the chain for different distances between cavities. $L = 1$ mm. h=5 μm $\Delta y = 3h$(2.8456 ms), $4h$(2.7955 ms), $5h$(2.7729 ms), $6h$(2.7688 ms). $E_* = 350$ kV/cm. α=20 cm/(kV·ms). Lattice size is 200 × 200.

current density $\mathbf{j} = \sigma\cdot\mathbf{E}$ are nonzero only within gas inclusions during the partial discharges. The time-implicit finite-difference scheme is used to solve this system of equations [13,19,20]. The boundary conditions are $\phi = 0$ at $y = 0$ and $\phi = V$ at $y = L$. The periodic boundary conditions are used in x direction. The permittivity of the condensed dielectrics is assumed to be equal to $\varepsilon = 2$.

The values of electric field strength inside uncharged gas cavities are greater than the electric field strength in condensed dielectrics $E = kE_0$, where $E_0 = V/L$ is the uniform electric field in dielectric far from the cavity. Here, k is the geometric factor depending on the dimensionality of space, the shape of cavity and relative permittivity of condensed dielectric.

The polarization of a dielectric near the cavity increases the value of electric field in a cavity. For a single isolated spherical cavity, the well-known formula $E = E_0 3\varepsilon/(2\varepsilon + 1)$ is valid [23] that gives us the value $k = 6/5$ for $\varepsilon = 2$. For a single infinite cylindrical cavity perpendicular to the electric field one can obtain $E = E_0 2\varepsilon/(\varepsilon + 1)$ that estimates the value $k = 4/3$ for $\varepsilon = 2$. Unfortunately, the analytic formulas for cubic (in 3D case) and square cavity (in 2D case) are absent.

Some estimations can be obtained for these cavities in approximation of "frozen" polarization of dielectrics. The electric field strength in the center of single cubic cavity is exactly equal to the value of uniform field inside a single isolated spherical cavity $E = E_0(\varepsilon + 2)/3$. For $\varepsilon = 2$, we have the value $k = 4/3$. By analogy, for two-dimensional case, the field strength in the center of square is exactly equal to the value of uniform field inside a circle (an infinite cylindrical cavity perpendicular to the electric field) $E = E_0(\varepsilon + 1)/2$. For $\varepsilon = 2$, we have the value $k = 3/2$.

In our two-dimensional calculations with a single square cavity we obtain $k \approx 1.28$ at $\varepsilon = 2$ that is close to the value $k = 1.5$ for "frozen" polarization and $k = 4/3 \approx 1.33$ for an infinite cylindrical cavity for real polarization.

For the chain located in the center of the interelectrode gap, the values of the electric field inside the first and the last cavities are greater than the values in the cavities at a central region of the chain (Fig. 3).

In this work, the linearly increasing voltage $V = \gamma t$ is applied to the electrodes. For the central position of the chain in the gap between electrodes, the difference in values of electric field inside the cavities at the ends of the chain and in its central region is greater for relatively small spacing Δy, than for large spacing (Fig. 3). The distribution depends also on the position of the chain in the gap between electrodes. If the chain is located near the electrode, the distribution of inner electric field in cavities along the chain becomes asymmetric (Fig. 4).

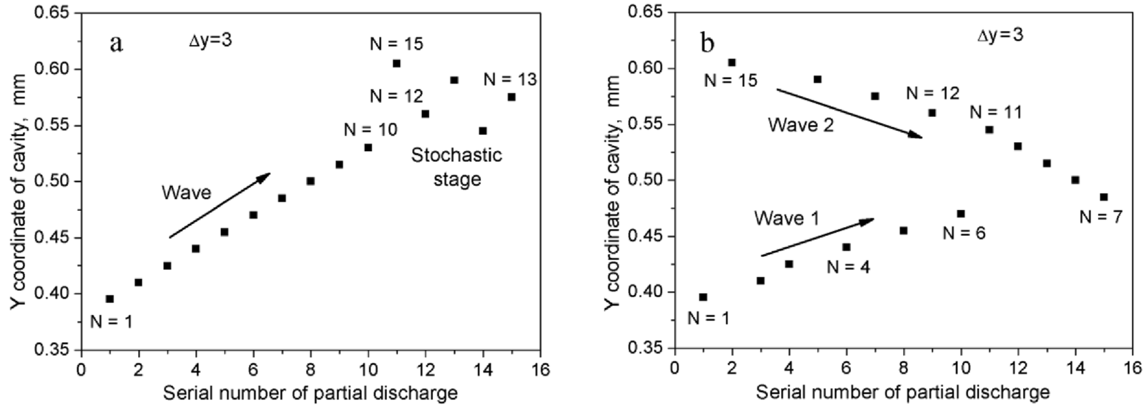

Fig. 6. The sequence of partial discharges in the cavities. (a) One wave of PDs and the stochastic stage. (b) Two waves of PDs from the both edges of the chain. L = 1 mm. E_* = 350 kV/cm. Δy = 3h. h=5 μm α=1 cm/(kV·ms). Lattice size is 200 × 200.

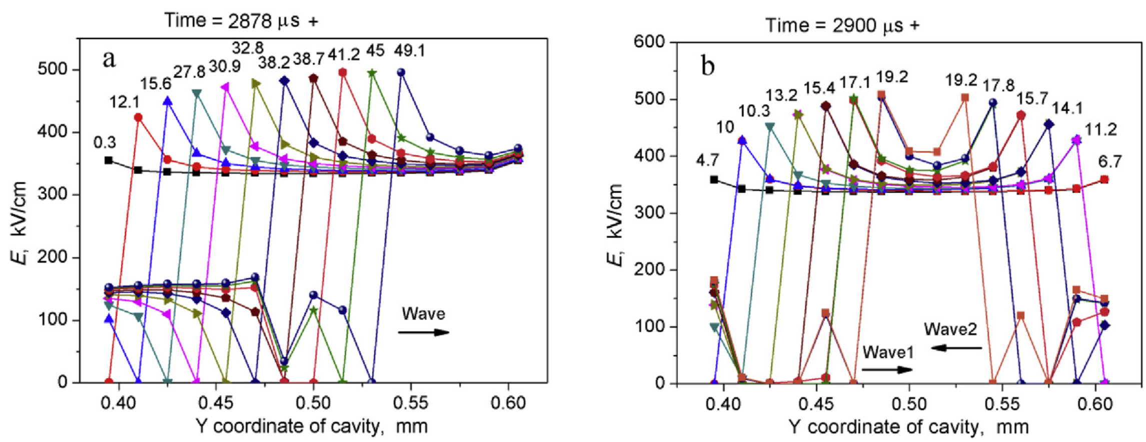

Fig. 7. The electric field values inside the cavities as the waves of PDs propagate along the chain. (a) One wave of PDs. (b) Two waves of PDs from the both edges of the chain. L = 1 mm. E_* = 350 kV/cm. Δy = 3h. h=5 μm α=1 cm/(kV·ms). Lattice size is 200 × 200.

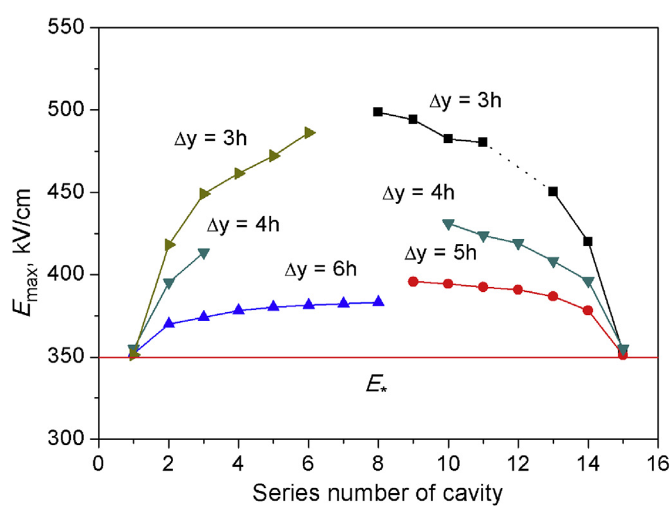

Fig. 8. The maximal values of electric field ahead of the front of the waves of PDs at different distances between cavities Δy. L = 1 mm. h=5 μm E_* = 350kV/cm. α=20 cm/(kV·ms). Lattice size is 200 × 200.

4. Partial discharges in a chain (symmetrical position)

For linearly increasing voltage $V = \gamma t$, the values of the electric field strength inside the first and the last cavities in the chain at a certain moment of time become somewhat greater than the threshold value E_* (Fig. 5). After that, the probabilities of micro-discharge f inside these gas inclusions during time step Δt become noticeable. The rate of voltage rise is chosen equal to γ=10 kV/ms. The smaller the spacing Δy, the later the electric field in the cavities at the edges of the chain reaches the critical value (Fig. 3). Hence, the micro-discharge occurs later (at 2.8456 ms for Δy = 3h) (Fig. 5). For symmetrical position of a chain, the PD waves can be initiated as at first cavity (Fig. 6a) or at last cavity. Occasionally, two PD waves can be observed together from the both edges of the chain (Fig. 6b).

The first example of simulation is shown in Figs. 6a and 7a. After the partial discharge in the first gas cavity (N = 1), this cavity becomes conductive. Since short time of charge relaxation, the value of electric field strength in the neighbor cavity (N = 2) becomes around 430 kV/cm that is considerably greater than the threshold value E_*. Hence, the probability of partial discharge in this gas inclusion increases significantly, and after short period, the micro-discharge occurs here. Then, the process repeats for the next gas cavity (N = 3) and so on. As a result, the wave of partial discharges propagates along the chain of cavities (Figs. 6a and 7a). This special mode of propagation of partial discharges in the chain of cavities can be named as a "relay-race" mechanism.

Occasionally, the process with two PD waves that initiated from the both edges of the chain is possible (Figs. 6b and 7b).

As the wave of PDs propagates along the chain, the electric field strength increases inside the remaining cavities. After the moment t=2927 μs all values of electric field strength inside these cavities become greater than the critical value E_* (Fig. 7a), and the sequence of partial discharges can become stochastic (Fig. 6a).

The "relay-race" mechanism of propagation of partial discharges

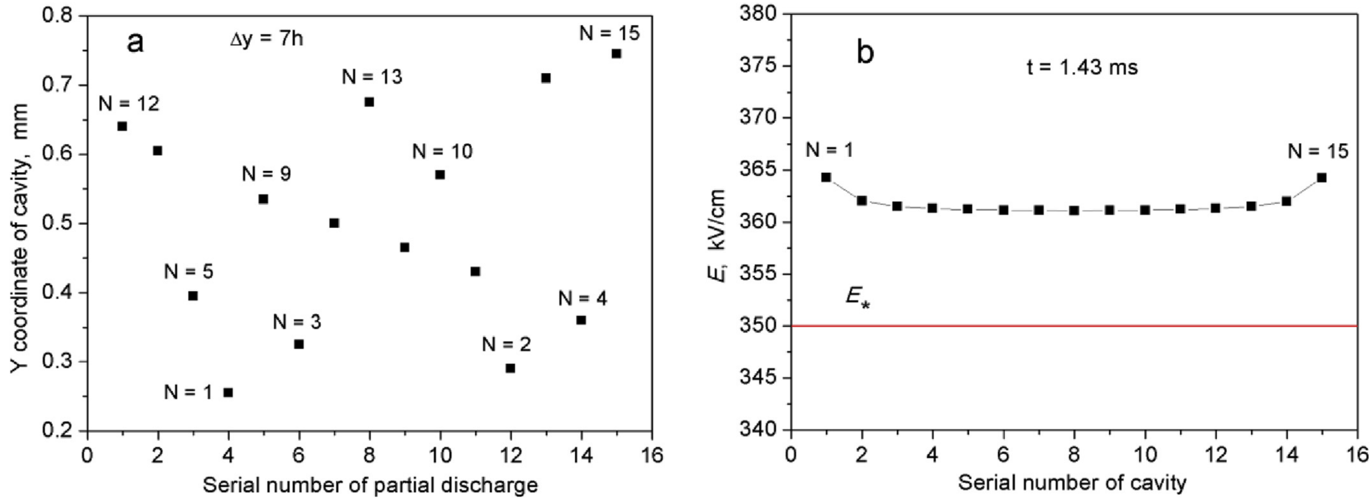

Fig. 9. (a) The stochastic regime of partial discharges in the chain of cavities. (b) The values of electric field inside cavities before the first PD (t = 1.43ms). L = 1 mm. E_* = 350 kV/cm. γ = 20 kV/s. $\Delta y = 7h$. h=5 μm α=1 cm/(kV·ms). Lattice size is 200 × 200.

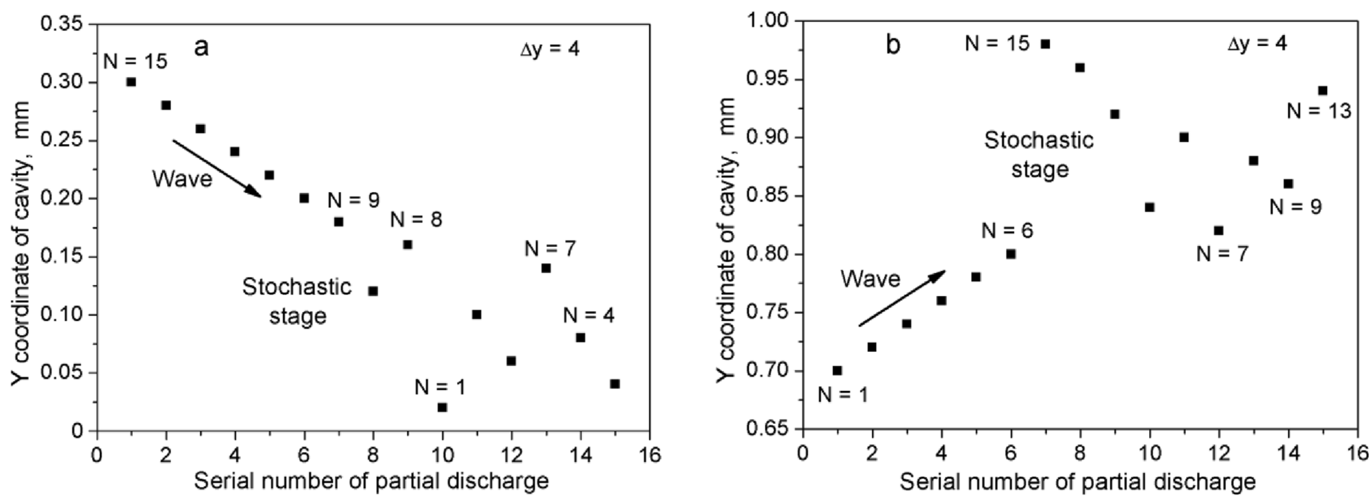

Fig. 10. The sequence of partial discharges in cavities. Wave of PDs and stochastic stage for the chains shifted down (a) and shifted up (b). L = 1 mm. E_* = 350kV/cm. $\Delta y = 4h$. h=5 μm α=20(a) and 5(b) cm/(kV·ms). Lattice size is 200 × 200.

can be realized if the dependence $r(E)$ is sharp enough and the distances between the cavities Δy are relatively small. After the microdischarge inside a cavity, the electric field in the neighbor cavity increases significantly and becomes considerably greater than the threshold value E_*. The maximal values of electric field strength ahead of the front of the PD waves for different distances between cavities Δy are shown in Fig. 8. For relatively small spacing Δy, the enhancing of electric field in the neighbor cavity is more pronounced. The PD waves can be initiated as at first cavity and at last cavity in the chain. The example of two waves of partial discharges is shown in Fig. 8 (the simulation for $\Delta y = 4h$).

The sequence of PDs in the chain of cavities becomes stochastic at a relatively weak dependence $r(E)$ on the electric field strength or if the distances Δy is relatively large and the mutual influence of cavities on each other is weak. For distances between cavities $\Delta y = 7h$, the sequence of the partial discharges in the chain occurs in the stochastic regime (Fig. 9a). In this case, the mutual influence of cavities on each other is not enough, and the values of electric field strength inside cavities have the close values (Fig. 9b).

5. Partial discharges in a chain located closer to electrode (asymmetrical position)

The examples of distribution of electric field strength for the asymmetrical positions of the chain are shown in Fig. 4. For the position of the chain closer to the lower electrode, the maximal value of electric field before the first partial discharge is inside the cavity number 15. For the position of the chain near the upper electrode, the maximal value of electric field is inside the cavity number 1. The waves of partial discharges for these both cases are demonstrated in Fig. 10.

6. Conclusion

The possibility of propagation of a wave of partial discharges in a linear chain of gas inclusions in solid dielectrics by means of "relay-race" mechanism is shown for linearly increasing voltage. The occurrence of a PD wave is possible if there is a significant mutual influence of partial discharges on the electric field strength in neighboring cavities. This is possible if the distance between cavities Δy is relatively small. In this case, the wave of partial discharges can propagate along the chain of cavities. The sequence of partial discharges has a completely stochastic character if the mutual influence of partial discharges in cavities on neighbor cavities is weak or at a relatively weak dependence $r(E)$ on electric field. The PD waves can be initiated as in the first cavity or in the last cavity in the chain if the electric field strength there exceeds the threshold value. The regular wave of partial discharges occurs if the electric field values in all inner cavities are less than the threshold value. Occasionally, there is a simultaneous initiation of two waves of partial discharges from both edges of the chain. If the electric field values in all cavities become above the threshold value, the sequence of partial discharges in the chain becomes stochastic.

There are several possible ways that can be used to observe this "relay-race" mechanism in experiments. The purpose of this article is to attract the attention of experimentalists to this phenomenon. The linear chain of bubbles was used as a model system, for which the waves can be observed in a clear form. In reality, the bubbles are distributed rather irregularly; nevertheless they can form the local nonlinear chains. Such a case can be also simulated easily, but it is more complicated for understanding of the process and has no independent interest from the physical point of view.

The experimental study of the phenomenon of "relay-race" propagation of partial discharges wave can give the important information about the dependence of the probability function $r(E)$.

Acknowledgments

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References

[1] F.H. Kreuger, E. Gulski, A. Krivda, Classification of partial discharges, IEEE Trans. Electr. Insul. 28 (1993) 917–931.
[2] L. Niemeyer, A generalized approach to partial discharge modeling, IEEE Trans. Dielectr. Electr. Insul. 2 (1995) 510–528.
[3] B. Fruth, L. Niemeyer, The importance of statistical characteristics of partial discharge data, IEEE Trans. Electr. Insul. 27 (1) (1992) 60–69.
[4] O. Lesaint, G. Massala, Positive streamer propagation in large oil gaps, IEEE Trans. Dielectr. Electr. Insul. 5 (3) (1998) 360–370.
[5] O. Lesaint, A. Saker, P. Gournay, R. Tobazeon, J. Aubin, M. Mailhot, Streamer propagation and breakdown under ac voltage in very large oil gap, IEEE Trans. Dielectr. Electr. Insul. 5 (3) (1998) 351–359.
[6] A. Peschot, C. Poulain, N. Bonifaci, O. Lesaint, Electrical breakdown voltage in micro- and submicrometer contact gaps (100 nm – 10 μm) in air and nitrogen, Proceedings of the IEEE Holm Conference on Electrical Contacts, San Diego, USA, 2015, pp. 280–286.
[7] Y. Suwarno, F. Suzuoki, T. Komori, Mizutani, Partial discharges die to electrical treeing in polymers: phase-resolved and time-sequence observation and analysis, J. Phys. D Appl. Phys. 29 (11) (1996) 2922–2931.
[8] A. Gemant, W. Von Philipoff, Die Funkenstrecke mit Vorkondensator, Zeitschrift für Technische Physik 13 (1932) 425–430.
[9] M. Hikita, K. Yamada, A. Nakamura, T. Mizutani, A. Oohasi, M. Ieda, Measurements of partial discharges by computer and analysis of partial discharge distribution by the Monte Carlo Method, IEEE Trans. Electr. Insul. 25 (3) (1990) 453–468.
[10] A. Cavallini, G.C. Montanari, Effect of supply voltage frequency on testing of insulation system, IEEE Trans. Dielectr. Electr. Insul. 13 (1) (2006) 111–121.
[11] E. Lemke, A critical review of partial-discharge models, IEEE Electr. Insul. Mag. 28 (6) (2012) 11–16.
[12] K. Wu, Y. Suzuoki, L.A. Dissado, The contribution of discharge area variation to partial discharge patterns in disk-voids, J. Phys. D Appl. Phys. 37 (13) (2004) 1815–1823.
[13] A.L. Kupershtokh, D.I. Karpov, D.A. Medvedev, C.P. Stamatelatos, V.P. Charalambakos, E.C. Pyrgioti, D.P. Agoris, Stochastic models of partial discharge activity in solid and liquid dielectrics, IET Sci. Meas. Technol. 1 (6) (2007) 303–311.
[14] A.L. Kupershtokh, C.P. Stamatelatos, D.P. Agoris, Simulation of partial discharge activity in solid dielectrics under AC voltage, Tech. Phys. Lett. 32 (8) (2006) 680–683.
[15] A.L. Kupershtokh, D.I. Karpov, Simulation of waves of partial discharges in a chain of gas inclusions located in condensed dielectrics, J. Phys.: Conf. Ser 754 (10) (2016) 102006.
[16] P. Gournay, O. Lesaint, On the gaseous nature of positive filamentary streamers in hydrocarbon liquids. II: propagation, growth and collapse of gaseous filaments in pentan, J. Phys. D Appl. Phys. 27 (10) (1994) 2117–2127.
[17] N.Yu Babaeva, D.V. Tereshonok, G.V. Naidis, B.V. Smirnov, Initiation of breakdown in strings of bubbles immersed in transformer oil and water: string orientation and proximity of bubbles, J. Phys. D Appl. Phys. 49 (2) (2016) 025202.
[18] P. Biller, Fractal streamer models with physical time, Proceedings of the 11th Conference on Conduction and Breakdown in Dielectric Liquids, 1993, pp. 199–203 IEEE No. 93CH3204-5, Baden-Dättwil, Switzerland.
[19] D.I. Karpov, A.L. Kupershtokh, Models of streamers growth with "physical" time and fractal characteristics of streamer structures, IEEE No. 98CH36239, Conference Record of the 1998 IEEE Int. Symposium on Electrical Insulation, vol. 2, 1998, pp. 607–610. Arlington, USA.
[20] A.L. Kupershtokh, V. Charalambakos, D. Agoris, D.I. Karpov, Simulation of breakdown in air using cellular automata with streamer to leader transition, J. Phys. D Appl. Phys. 34 (6) (2001) 936–946.
[21] A.L. Kupershtokh, C. Stamatelatos, D.P. Agoris, Stochastic model of partial discharge activity in liquid and solid dielectrics, Proceedings of the 15th IEEE Int. Conf. On Dielectric Liquids, Coimbra, Portugal, 2005, pp. 135–138.
[22] M. Hayashi, Statistical fluctuation of discharge, Oyo Butsuri (The Japan Soc. of Appl. Phys.) 40 (1971) 1133–1138.
[23] J.A. Stratton, Electromagnetic Theory, McGraw-Hill, New York, 1941.