

Use of the lattice Boltzmann method for simulations of heating a "plasma" in channels and vapor-gas cavities at electrical discharges in liquid dielectrics

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Abstract. The hybrid physico-mathematical model based on the two-component lattice Boltzmann method is proposed for computer simulations of "plasma" channels and vapour-gas cavities at electrical discharges in liquid dielectrics. In this model, two different states of matter are considered separately. The first component describes the flows of a dielectric liquid. The second component describes the "plasma" substance during its heating inside the channel.

1. Introduction

During the electrical discharge in dielectric liquids, the conductive "plasma" channels arise in a liquid [1] that can be simulated by the lattice Boltzmann equations method (LBM, [2,3]). An analysis is carried out of the possibility of modeling using the LBM the evolution of these "plasma" channels, as well as the partial electric discharges within vapor-gas cavities in liquid dielectrics. The problem is that the parameters of the substance in the channels or cavities (temperature, etc.) differs many times from their values in the surrounding dielectric liquid. Therefore, it is difficult to ensure the stability of calculations for strong jumps in temperature and density at the interface. This problem arises for all methods of numerical simulations both for finite-difference methods and for LBM.

In this paper, a hybrid physico-mathematical model describing the flow of liquid dielectrics with phase transitions at the boundaries of "plasma" channels is constructed. The basis for this is the variant of the LBM described in detail in [4,5]. The lattice Boltzmann method has been widely exploited and it is quite competitive with the traditional methods of computational fluid dynamics. The LBM has considerable advantages, especially for multiphase and multicomponent flows with complex topology.

2. Lattice Boltzmann Equation Method

The lattice Boltzmann equation method treats fluid flows as an ensemble of pseudo-particles that can move on a regular space lattice. In this method, the values of distribution functions $N_k(\mathbf{x}, t)$ (pseudo-particles) transfer during time step Δt from a node to neighbor nodes $\mathbf{e}_k = \mathbf{c}_k \Delta t$ along the characteristics. For this purpose, the several discrete finite sets of particle velocities \mathbf{c}_k for ($k = 0, 1, \dots, b$) were proposed [6]. For the one-dimensional three-speed model D1Q3, $b = 3$; for the two-dimensional nine-speed model D2Q9 on square lattice, $b = 8$; and for the three-dimensional nineteen-speed model D3Q19 on cubic lattice, $b = 18$.



The evolution equation for the distribution functions has the form

$$N_k(\mathbf{x} + \mathbf{c}_k \Delta t, t + \Delta t) = N_k(\mathbf{x}, t) + \Omega_k(\{N\}) + \Delta N_k(\mathbf{x}, t), \quad (1)$$

where Ω_k is the collision operator, and ΔN_k is the change of the distribution functions because of the action of the external and internal body forces.

The density ρ and the velocity \mathbf{u} of a fluid in a node are calculated as

$$\rho = \sum_{k=0}^b N_k, \quad (2)$$

$$\rho \mathbf{u} = \sum_{k=1}^b \mathbf{c}_k N_k. \quad (3)$$

The collision operator in the BGK form can be written as

$$\Omega_k = (N_k^{eq}(\rho, \mathbf{u}) - N_k(\mathbf{x}, t)) / \tau, \quad (4)$$

where τ is the dimensionless single relaxation time. This collision operator simulates the tendency of the distribution functions to their equilibrium values [7]

$$N_k^{eq}(\rho, \mathbf{u}) = \rho w_k \left(1 + \frac{\mathbf{c}_k \mathbf{u}}{\theta} + \frac{(\mathbf{c}_k \mathbf{u})^2}{2\theta^2} - \frac{\mathbf{u}^2}{2\theta} \right). \quad (5)$$

Here, $\theta = (h/\Delta t)^2/3$ is the "kinetic" temperature of LBE pseudo-particles. The weight coefficients are equal to $w_0 = 2/3$ and $w_{1,2} = 1/6$ for the model D1Q3, $w_0 = 4/9$, $w_{1-4} = 1/9$, $w_{5-8} = 1/36$ for the model D2Q9, and $w_0 = 1/3$, $w_{1-6} = 1/18$, $w_{7-18} = 1/36$ for the model D3Q19 [6]. The kinematic viscosity of fluid for all these three models is equal to $\nu = \theta(\tau - 1/2)\Delta t$.

To implement the body forces in the LBM, the Exact difference method (EDM) was proposed in [8,9]. The corresponding change of distribution functions during the time step has the form

$$\Delta N_k(\mathbf{x}, t) = N_k^{eq}(\rho, \mathbf{u} + \Delta \mathbf{u}) - N_k^{eq}(\rho, \mathbf{u}). \quad (6)$$

Here, $\mathbf{u} + \Delta \mathbf{u} = \mathbf{u} + \mathbf{F}\Delta t/\rho$ is the value of the velocity after the action of the total force \mathbf{F} on a node \mathbf{x} . If the body forces are present, the physical value of fluid velocity is not equal to the velocity \mathbf{u} used in LBM calculations (3), but should be recalculated at a half time step [10] as $\mathbf{u}_* = \mathbf{u} + \Delta \mathbf{u}/2$.

Depending on specific problems, the periodic boundary conditions, boundary conditions without sliding or sliding on the solid surface, and boundary conditions of inflow/outflow can be used.

In order to simulate the condensation of vapor into liquid and the reverse process, an attractive part of the intermolecular potential should be taken into account. For this purpose, it was proposed to introduce into the LBM the special attractive forces between the substances in neighbor nodes [11]. These forces related to the densities in nodes. Later, the pseudopotential model was proposed in which the total force acting on the fluid in a node was introduced as $\mathbf{F} = -\nabla U$ [12]. Here, $U = P(\rho, T) - \rho\theta$ is the pseudopotential that was defined using the specific equation of state for the fluid.

The numerical approximation of the gradient of the pseudopotential is very important for correct describing the phase coexistence curve. In [4,5], the function $\Phi = \sqrt{-U}$ was specially introduced. In this case, the total force can be written in the equivalent form as $\mathbf{F} = 2A\nabla(\Phi^2) + (1-2A)2\Phi\nabla\Phi$. The so-called "combined" finite-difference approximation of this formula was proposed in [4,5] that can be written in the vector form as

$$\mathbf{F}(\mathbf{x}) = \frac{1}{\alpha h} \left[A \sum_k \frac{G_k}{G} \Phi^2(\mathbf{x} + \mathbf{e}_k) \mathbf{e}_k + (1-2A)\Phi(\mathbf{x}) \sum_k \frac{G_k}{G} \Phi(\mathbf{x} + \mathbf{e}_k) \mathbf{e}_k \right]. \quad (7)$$

The values of numerical coefficient α are equal to 1, 3/2 and 3 for the models D1Q3, D2Q9 and D3Q19, respectively [5]. The coefficients G_k are equal to G for basic directions and to $G/4$ and $G/2$ for diagonal directions for the models D2Q9 and D3Q19, respectively [4,5,13]. This "combined"

approximation (7) is more stable than two finite-difference approximations (local and “mean” value [4,5,13]) taken separately. The free parameter A can be tuned for each specific equation of state. For the van der Waals equation of state, its optimal value is $A = -0.152$ [4]. This equation can be written in the reduced variables as

$$\tilde{P} = 8\tilde{\rho}\tilde{T}/(3 - \tilde{\rho}) - 3\tilde{\rho}^2. \tag{8}$$

Here, the reduced variables are $\tilde{P} = P/P_{cr}$, $\tilde{\rho} = \rho/\rho_{cr}$, $\tilde{T} = T/T_{cr}$, where P_{cr} , ρ_{cr} , T_{cr} are the pressure, density and temperature at the critical point.

For this pseudopotential approach, the density of fluid changes smoothly over several lattice nodes in a thin transition layer between liquid and vapor. In this method, the surface tension at interphase boundaries is present. It is known that the possibility of simulations using the algorithm of LBM is limited by the stability condition obtained in [9]

$$\tilde{c}_s = \sqrt{\tilde{k} \left(\frac{\partial \tilde{P}}{\partial \tilde{\rho}} \right)_T} < 1 + \tilde{\theta}. \tag{9}$$

Here, \tilde{c}_s is the hydrodynamic Courant number calculated from an equation of state of fluid, $\tilde{\theta}$ is the dimensionless "kinetic" temperature of LBE pseudo-particles. Hence, the stability of LBM depends on the temperature and on the dimensionless parameter $\tilde{k} = P_{cr}\Delta t^2 / (\rho_{cr}h^2)$.

The CUDA technology is used to parallelize the calculations on GPUs (two NVIDIA Titan-XP).

3. One-component model

In the simplest version of the model, there is only one component in which a liquid-vapor phase transition is possible in accordance with the equation of state. This model describes two phases: liquid and "vapor". The "vapor" can be heated to a much higher temperature than the liquid dielectric that allows us to interpret "vapor" as "plasma".

The heating and expansion of the "plasma" channel in a liquid dielectric with a uniform temperature distribution inside the discharge channel is simulated. Test calculations are performed using a one-dimensional model DIQ3 of lattice Boltzmann equations method. The periodic boundary conditions are used. Heating begins after a certain time t_0 for establishing the initial state of rest at $\tilde{T}_0 = 0.6$. The prescribed linear dependence of temperature on time $d\tilde{T}/dt \approx 0.001$ is used in this case. However, in this model, the highly excessive flows of material at the interface between the liquid dielectric and the discharge channel are changed considerably as the temperature of the "plasma" increases (figure 1, curve1). This makes it impossible to correctly simulate the process.

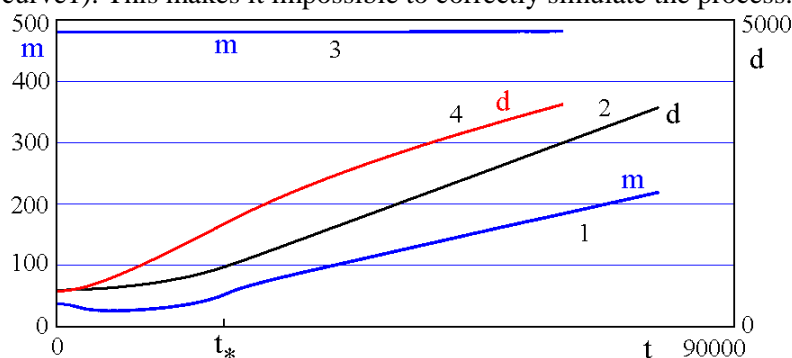


Figure 1. Time dependences of mass of “plasma” in the channels (curves 1 and 3) and the diameter of the channels (curves 2,4). Curves 1,2 are for the one-component model. Curves 3,4 are for the hybrid two-component model. The moment of termination of heating is t_* .

4. Hybrid two-component model

To avoid this parasitic fluxes of mass, the two-component physico-mathematical model based on the

LBM is constructed. In this model, two different states of matter are considered separately. The first component describes the flows of a liquid dielectric. The second component describes the substance in the discharge channel ("plasma") during its heating. In this case, the important property of the LBM to capture the interphase boundaries is also preserved. For two-component model, the mass of material in the channel is practically conserved (figure 1, curve 3).

For the dielectric liquid, the van der Waals equation of state (8) is used. However, any other equations of state that allow a liquid-vapor phase transition can be used in this approach. The equation of state of an ideal gas is used as a model for the "plasma" in the discharge channel. Obviously, this equation of state of the material in the discharge channels or cavities should not have phase transitions. In order for these two equations of state to conform to each other in the gas limit $\tilde{\rho} \ll 1$, the compressibility factor $Z = P_{cr}V_{cr}/(RT_{cr})$ must be the same for both cases. For the van der Waals equation of state, the compressibility factor is equal to $3/8=0.375$. Hence, for ideal gas in the channel, the corresponding equation of state in reduced variables is used in the form $\tilde{P} = 8\tilde{\rho}\tilde{T}/3$.

5. Simulations for hybrid two-component model

The heating and expansion of the "plasma" with a uniform temperature distribution inside the single discharge channel in a liquid dielectric is simulated. Test calculations are performed using a one-dimensional model D1Q3 of the lattice Boltzmann equations method. The periodic boundary conditions are used in x direction.

To reduce the solubility of a gas ("plasma") in a liquid, the repulsive forces acting on the substance of each component at a node \mathbf{x} from the substance of another component that is present at neighbor nodes $(\mathbf{x} + \mathbf{e}_k)$ are simulated. The total forces acting on the substance of components s at a node \mathbf{x} from the component σ have the form

$$\mathbf{F}^s(\mathbf{x}) = \psi[\rho^s(\mathbf{x})] \sum_k B_k \psi[\rho^\sigma(\mathbf{x} + \mathbf{e}_k)] \mathbf{e}_k, \quad (10)$$

Here, $\psi(\rho)$ is the increasing function that depends on the density of the corresponding component. The coefficients B_k determine the degree of immiscibility of the components.

For this hybrid one-dimensional model, the temperature in the discharge channel increases from the initial value $\tilde{T}_0 = 0.6$ to some finite value $\tilde{T} \approx 23$ with the rate $d\tilde{T}/dt \approx 0.001$ and then remains constant. In this case, the initial density of the liquid on the coexistence curve is $\tilde{\rho}_0 = 2.31$ for the van der Waals equation of state. In accordance with the stability condition (9), the upper limit of temperature in the "plasma" channels is $\tilde{T} \approx 23$ at the dimensionless parameter $\tilde{k} = 0.005$. The corresponding pressure is $\tilde{p} \approx 6.3$.

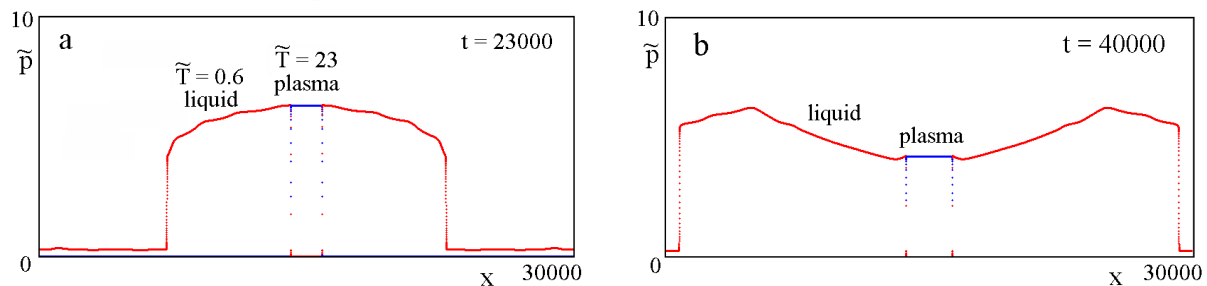


Figure 2. Pressure distributions in the neighborhood of the "plasma" channel (a) at the end of heating $t = 23000$ ($\tilde{T} \approx 23$) and (b) in the process of the further channel expansion $t = 40000$.

The pressure distribution in the neighborhood of the discharge channel after the moment of termination of heating $t_* = 23000$ is shown in figure 2a. The temperature in the channel is $\tilde{T} \approx 23$ at

this moment.

When the channel expands in a dielectric liquid in one-dimensional case, a shock wave is formed, followed by a compression wave. As the expansion of the channel continues after the termination of heating, the pressure inside the channel decreases, and a rarefaction wave is formed and propagates into the liquid (figure 2).

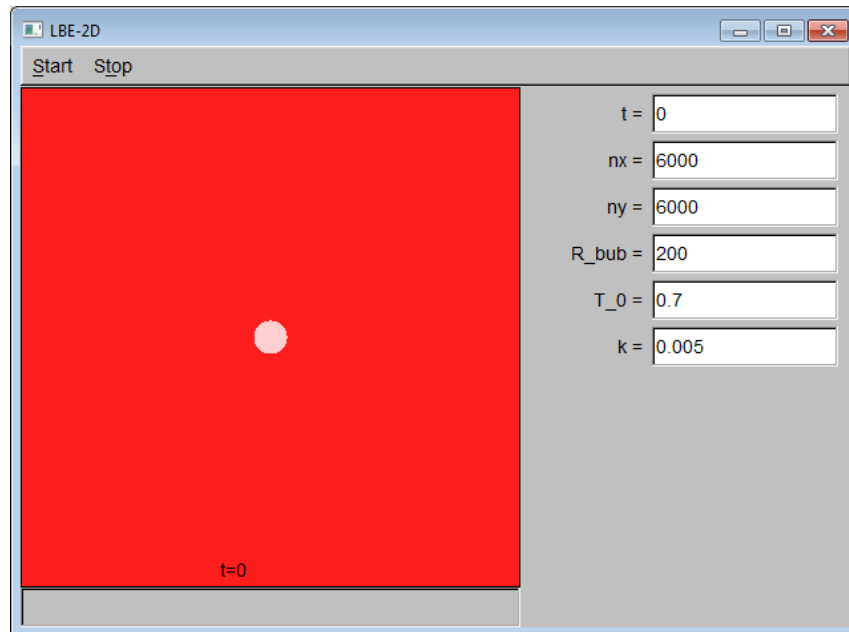


Figure 3. Cross section of the "plasma" channel in liquid dielectric for two-dimensional two-component model. Initial radius of the channel is 200 lattice units. $\tilde{T}_0 = 0.7$. Lattice is 6000×6000.

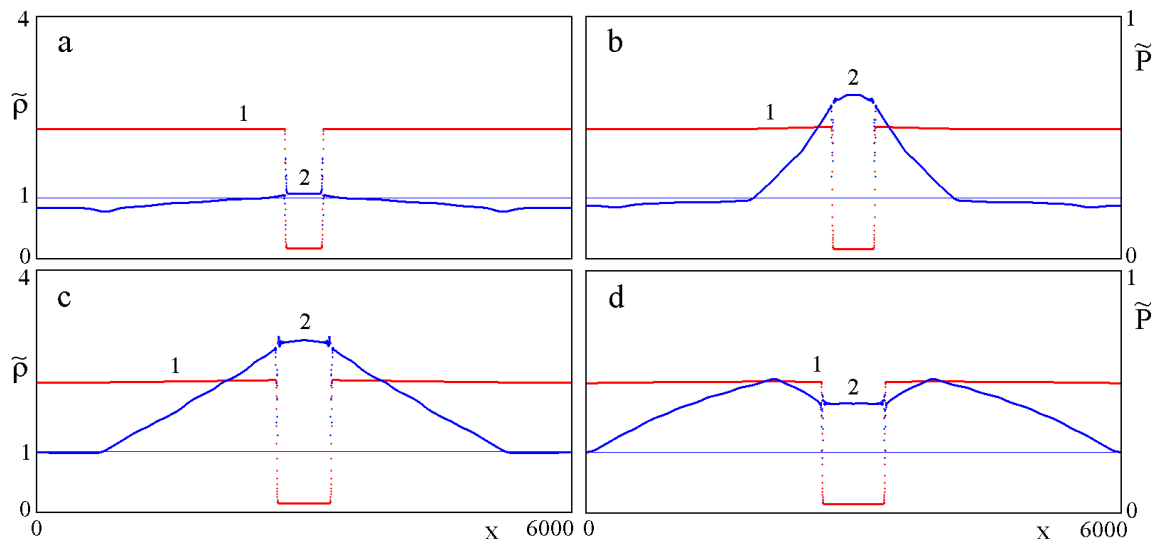


Figure 4. The distributions of density of matter $\tilde{\rho}$ (curves 1) along the x direction in central cross section of the system and the pressure distributions (curves 2) in the neighborhood of the "plasma" channel. $t = 10000$ (a), 15000 (b), 20000 (c), 22500 (d).

The model D2Q9 is used in two-dimensional simulations (figure 3). The periodic boundary conditions are used in x and y directions. The initial distributions of component densities are established for some time ($t = 10000$ time steps) before the process of heating the "plasma" to obtain the initial state of the system (figure 4a) that is close to the stationary state. In this case, the initial

density of the liquid on the coexistence curve is $\tilde{\rho}_0 = 2.14$ for the van der Waals equation of state.

For two-dimensional case, the prescribed temperature in the discharge channel increases linearly from the initial value $\tilde{T}_0 = 0.7$ to some finite value and then begins to decrease. The rates of the temperature increase and decrease are chosen as $d\tilde{T}/dt \approx 0.001$ and $d\tilde{T}/dt \approx -0.002$, respectively.

After the temperature in the gas-vapor channel begins to increase, the divergent compression wave is generated in the liquid because of the channel expansion (figures 4b and 4c). After the temperature in the gas-vapor channel starts to decrease, the rarefaction wave is generated in the liquid (figure 4d).

6. Conclusion

The hybrid physico-mathematical model based on the two-component lattice Boltzmann method is proposed for computer simulations of “plasma” channels and vapour-gas cavities at electrical discharges in liquid dielectric. Two different states of the matter are considered separately. The first component describes the liquid dielectric. The second component describes the “plasma” during its heating inside the channel. This model allows one to reduce considerably the parasitic flows of material at the interface between the liquid dielectric and the “plasma” of the discharge channel.

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